PAPER SOLUTION

From Meerut

JEE MAIN

JAN	SHIFT
24	1st

2025

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JEE MAIN 2025 LIVE PAPER DISCUSSION

#Q. If $f(x)\frac{2^{x+2}+16}{2^{2x+1}+2^{x+4}+32}$, then $f\left(\frac{1}{15}\right)+f\left(\frac{2}{15}\right)+\ldots+f\left(\frac{59}{15}\right)=$

(A) $\frac{59}{4}$

(B) $\frac{25}{8}$

(C) 15

(D) 20

$$f(x)=\frac{2^{x+2}+16}{2^{2x+1}+2^{x+4}+32}$$

$$=\frac{4\left[2^{x}+4\right]}{2\left[2^{2x}+8.2^{x}+16\right]}$$

$$f(x)=\frac{4(2^{x}+4)}{2(2^{x}+4)^{2}}$$

$$f(x)=\frac{2}{2^{x}+4}$$

$$f(x) = \frac{2}{2^x + 4}$$ ✓

$$f(4-x) = \frac{2}{2^{4-x} + 4}$$

$$= \frac{2}{\frac{16}{2^x} + 4}$$

$$= \frac{2^x}{8 + 2.2^x} = \frac{2^x}{2(2^x+4)}$$ ✓

$$f(x) + f(4-x) = \frac{4+2^x}{2(2^x+4)} = \frac{1}{2}$$

$$f\left(\frac{1}{15}\right) + f\left(\frac{2}{15}\right) + \ldots + f\left(\frac{59}{15}\right)$$

$f(2)$

$$\frac{29}{2} + \frac{1}{4} = \frac{59}{4}$$

Ans. (A)

#Q. Image of a circle about the line $2x - 3y + 5 = 0$ is $x^2 + y^2 - 2x + 4y - 4 = 0$. A line is drawn its centre O parallel to x axis which touch the circle at A. The point A lies at the right side of the centre. An arc of angle 60^0 is drawn from A to a point (α, β) on the circle such that $\beta < 4$. Find the value of $\beta - \sqrt{3}\alpha$

(A) 4

(B) $3\sqrt{3}$

(C) $4 - 3\sqrt{3}$

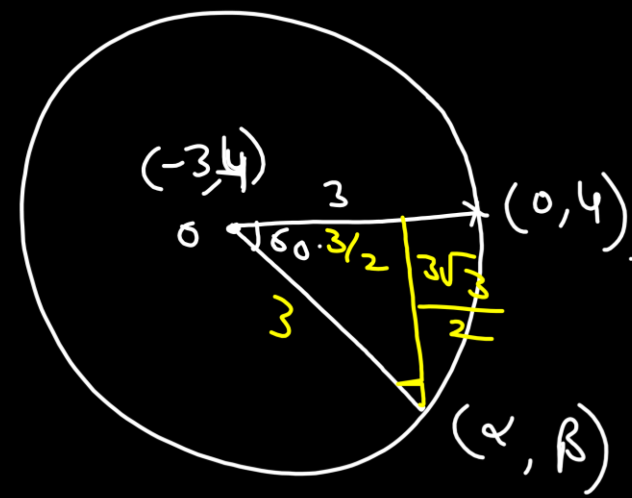
(D) $5 - 3\sqrt{3}$

$(1, -2)$, $r = 3$

$O(-3, 4)$

$\frac{x-1}{2} = \frac{y+2}{-3} = -2\left(\frac{2+6+5}{13}\right) = -2$

$x = -3, y = 4$



$\alpha = -\frac{3}{2}$, $\beta = 4 - \frac{3\sqrt{3}}{2}$

$\sqrt{3}\alpha = -\frac{3\sqrt{3}}{2}$

Ans. (A)

#Q. If S be the set of 10 distinct primes and let A be the set of product of two or more elements from the set S. If P = $\{(x, y): x \in S \text{ and } y \in A$ and y is ~~divided~~ divisible by $x\}$. Then n(P) is equal to

(A)

(B)

(C)

(D)

x: $S = \{2, 3, 5, 7, \ldots\}$

y: $A = \{2\times 3, 2\times 5, 2\times 7, 2\times 3\times 5, 2\times 5\times 7, \ldots\}$

$x = 2$, ${}^9C_1 + {}^9C_2 + {}^9C_3 + \ldots + {}^9C_9 = 2^9 - 1 = 511$

Total $= 511 \times 10 = 5110$

Ans. (5110)

#Q. If $l(m,n)=\int_0^1 x^{m-1}(1-x)^{n-1}dx, m,n>0$, then $l(9,14)+l(10,13)$ is equal to

$$B(m,n)=\frac{\Gamma m\,\Gamma n}{\Gamma(m+n)}$$

(A) $l(1,13)$

(B) $l(9,1)$

(C) $l(9,13)$ ✓

(D) $l(19,29)$

$$I(9,14)+I(10,13)$$

$$\frac{\Gamma 9\,\Gamma 14}{\Gamma 23}+\frac{\Gamma 10\,\Gamma 13}{\Gamma 23}=\frac{\Gamma 9\cdot 13\,\Gamma 13}{\Gamma 23}+\frac{9\,\Gamma 9\,\Gamma 13}{\Gamma 23}$$

$$=\frac{\Gamma 9\,\Gamma 13\cdot 22}{\Gamma 22\cdot 22}=I(9,13)$$

Ans. (C)

#Q. Mean of 10 numbers is 5.5 and $\sum_{i=1}^{10} x_i^2 = 371$

If the observations 4 and 5 are replaced by 6 and 8 respectively, then the new variance is

Old

$\sum x_i = 55$

$\sum x_i^2 = 371$

New:

$\sum x_i = 60$

$\sum x_i^2 = 371 - 16 - 25 + 36 + 64 = 430.$

$\sigma^2 = 43 - 36 = 7$

(A) 5

(B) 7 ✓

(C) 4

(D) 6

Ans. (B)

#Q. If $S_n = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \ldots$n terms. The sum of first six terms in A.P. With first term equal to $-p$ and common difference p is $\sqrt{2026.S_{2025}}$. The absolute value of difference between 20th and 15th term is A.P is

A

B

C

D

$S_n = \frac{1}{1\times 2} + \frac{1}{2\times 3} + \frac{1}{3\times 4} + \cdots + \frac{1}{n(n+1)}$

$= 1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \cdots + \frac{1}{n} - \frac{1}{n+1}$

$= 1 - \frac{1}{n+1}$

$S_n = \frac{n}{n+1}$, $S_{2025} = \frac{2025}{2026}$

45

$a = -5,\ d = 5$

$3[-p(2) + 5p] = 45$

$3 \times 3p = 45 \Rightarrow p = 5$

$a_{20} = -5 + 19 \times 5$

$a_{15} = -5 +$

$a_{20} - a_{15} = 5d = 25$

Ans. (25)

#Q. Find the product of all rational roots of equation $\left(x^2-9x+11\right)^2-(x-4)(x-5)=3$ is

(A) 21

(B) 14

(C) 7

(D) 28

$(x^2-9x+11)^2-(x^2-9x+20)-3=0$, $x^2-9x+20=t$

$(t-9)^2-t-3=0$

$t^2-19t+78=0$

$t=13, 6$

$x^2-9x+20=6$

$x^2-9x+14=0 \Rightarrow x=2, 7$

$x^2-9x+20=13$

$x^2-9x+7=0$, $D=81-28$

Ans. (B)

#Q. If $\frac{dy}{dx}+\left(\frac{x}{1+x^2}\right)y=\frac{\sqrt{x}}{\sqrt{1+x^2}}$; $y(0)=0$, then $y(1)$ will be

(A) $\frac{2}{3}$

(B) $\frac{2}{\sqrt{3}}$

(C) $\frac{\sqrt{2}}{3}$ ✓

(D) $\sqrt{\frac{2}{3}}$

$$I.F = e^{\frac{1}{2}\int\frac{2x}{1+x^2}dx} = e^{\frac{1}{2}\ln(1+x^2)} = e^{\ln\sqrt{1+x^2}} = \sqrt{1+x^2}$$

$$y\sqrt{1+x^2} = \int\sqrt{x}\,dx + c$$

$$y\sqrt{1+x^2} = \frac{2}{3}x^{3/2} + 0$$

$$y\sqrt{1+x^2} = \frac{2}{3}x\sqrt{x}$$

$x=1$

$$y\sqrt{2} = \frac{2}{3}$$

$$y = \frac{2}{3\sqrt{2}} = \frac{\sqrt{2}}{3}$$

Ans. (C)

$1+\tan^2(\tan^{-1}\alpha)$

#Q. If α *and* β are real numbers such that $\sec^2\left(\tan^{-1}(\alpha)\right) + \operatorname{cosec}^2\left(\cot^{-1}(\beta)\right) = 36$ and $\alpha + \beta = 8$, then $\left(\alpha^2 + \beta\right)$ is $(\alpha < \beta)$

A

B

C

D

$\sec^2(\tan^{-1}\alpha) = 1+\alpha^2$

$\operatorname{cosec}^2(\cot^{-1}\beta) = 1+\beta^2$

$2+\alpha^2+\beta^2 = 36$

$\alpha^2+\beta^2 = 34$

$25+9$

$\alpha = 3,\ \beta = 5$

$\alpha^2+\beta = 9+5 = 14$

Ans. (14)

$y = (x+2)^2 - 2, \; y = |x+2|$

#Q. The area of the region bounded by $S(x, y)$ such that $S = \{(x, y): x^2 + 4x + 2 \leq y \leq |x + 2|\}$ is (in sq. units)

(A) $\frac{24}{5}$

(B) 5

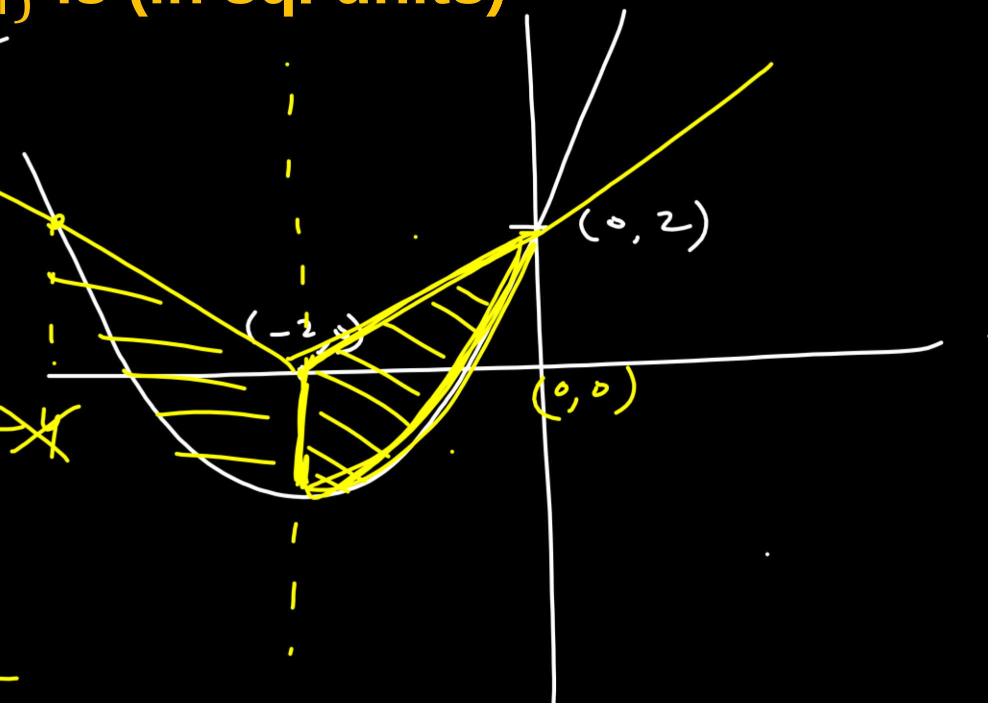
(C) $\frac{20}{3}$ ✓

(D) 7

$$2\int_{-2}^{0} \left[(x+2) - (x+2)^2 + 2\right] dx$$

$$= 2\left[\frac{(x+2)^2}{2} - \frac{(x+2)^3}{3} + 2x\right]_{-2}^{0}$$

$$= 2\left\{2 - \frac{8}{3} + 0 - (-4)\right\} = \frac{20}{3}$$



Ans. (C)

$P(\text{Sum of } 5) \to \frac{4}{36} = \frac{1}{9}$, $P(\text{sum of } 8) = \frac{5}{36}$

#Q. Two persons A and B throws a pair of dice alternatively. For A to win he should throw sum of 5 before B throws sum of 8. If A throws first, then the probability that A wins, is

(A) $\frac{8}{19}$

(B) $\frac{9}{19}$ ✓

(C) $\frac{8}{17}$

(D) $\frac{9}{17}$

$$P(A_W) = P(A_5) + P(\bar{A}_5)P(\bar{B}_8)P(A_5) + \cdots$$

$$= \frac{1}{9} + \frac{8}{9} \times \frac{31}{36} \times \frac{1}{9} + \frac{8}{9} \times \frac{31}{36} \times \frac{8}{9} \times \frac{31}{36} \times \frac{1}{9} + \cdots$$

$$= \frac{\frac{1}{9}}{1 - \frac{62}{81}} = \frac{9}{19}$$

Ans. (B)

#Q. $f(x) - 6f\left(\frac{1}{x}\right) = \frac{35}{3x} - \frac{5}{2}$. If $\lim_{x\to 0}\left(\frac{1}{\alpha x} + f(x)\right) = \beta$. Find $(\alpha + 2\beta)$

$3+1=4$

A

B

C

D

$x \to \frac{1}{x}$

$$f(x) - 6f\left(\frac{1}{x}\right) = \frac{35}{3x} - \frac{5}{2}$$

$$6\times\left[f\left(\frac{1}{x}\right) - 6f(x) = \frac{35x}{3} - \frac{5}{2}\right]$$

$$-35f(x) = \frac{35}{3x} + 6\times\frac{35x}{3} - \frac{5}{2}\times 7$$

$$f(x) = \left[-\frac{1}{3x} - 2x + \frac{1}{2}\right]$$

$$\lim_{x\to 0}\left[\frac{1}{\alpha x} - \frac{1}{3x} - 2x + \frac{1}{2}\right]$$

$$\lim_{x\to 0}\left[\frac{(3-\alpha)}{3\alpha x} - 2x + \frac{1}{2}\right]$$

$\alpha = 3$, $\beta = \frac{1}{2}$

Ans. (4)

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#Q. Evaluate $\lim_{x \to 0} \operatorname{cosec} x \left(\sqrt{2\cos^2 x + 3\cos x} - \sqrt{\cos^2 x + \sin x + 4}\right)$

(A) 0

(B) 1

(C) $\frac{1}{2\sqrt{5}}$

(D) $-\frac{1}{2\sqrt{5}}$

$$\lim_{x \to 0} \frac{1}{\sin x}\left(\frac{2\cos^2 x + 3\cos x - \cos^2 x - \sin x - 4}{\sqrt{2\cos^2 x + 3\cos x} + \sqrt{\cos^2 x + \sin x + 4}}\right)$$

$$\frac{1}{2\sqrt{5}} \lim_{x \to 0} \frac{-\sin x + \cos^2 x + 3\cos x - 4}{\sin x}$$

$$\frac{1}{2\sqrt{5}} \lim_{x \to 0} -\frac{\sin x}{\sin x} + \frac{(\cos^2 x - 1)(\cos x + 4)}{\sin x(\cos x + 1)}$$

$\downarrow$ 0

$$= -\frac{1}{2\sqrt{5}}$$

Ans. (D)

#Q. If $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = 3\hat{i} + \hat{j} - \hat{k}$ and $\vec{c}$ is coplanar with $\vec{a}$ and $\vec{b}$. Also $\vec{a} \cdot \vec{c} = 5$ and $\vec{c}$ is perpendicular to $\vec{b}$. Then $|\vec{c}|$ is

(A) 18

(B) 16

(C) $\frac{\sqrt{5}}{14}$

(D) $\sqrt{\frac{11}{6}}$

$\vec{b} \cdot \vec{c} = 0$

$\vec{c} = \lambda \vec{a} + \mu \vec{b}$

$\vec{b} \cdot \vec{c} = \lambda \vec{a} \cdot \vec{b} + \mu \vec{b} \cdot \vec{b}$

$0 = 2\lambda + \mu(11)$

$\vec{a} \cdot \vec{c} = \lambda \vec{a} \cdot \vec{a} + \mu \vec{a} \cdot \vec{b}$

$5 = 14\lambda + 2\mu$

$2\lambda = -11\mu$

$5 = -77\mu + 2\mu$

$5 = -75\mu$

$\mu = \frac{5}{-75} = -\frac{1}{15}$

$\lambda = \frac{11}{30}$

$\vec{c} = \lambda\vec{a} + \mu\vec{b}$.

1, 2, 3   3, 1, −1

$\vec{c} = \frac{11}{30}\vec{a} - \frac{1}{15}\vec{b}$.

$= \frac{11\vec{a} - 2\vec{b}}{30}$.

$= \frac{11\hat{i} + 22\hat{j} + 33\hat{k} - 2(3\hat{i} + \hat{j} - \hat{k})}{30}$

$= \frac{5\hat{i} + 20\hat{j} + 35\hat{k}}{30} = \frac{\hat{i} + 4\hat{j} + 7\hat{k}}{6}$.

$|\vec{c}| = \frac{\sqrt{1+16+49}}{6}$

$= \sqrt{\frac{66}{36}}$

$= \sqrt{\frac{11}{6}}$

Ans. (D)

#Q. If A is 3×3 matrix such that det(A) = 2. Then det $(adj(adj(adj(adjA))))$

$$\left(\left(\left(2^2\right)^2\right)^2\right)^2$$

$$2^{16}$$

(A) 2^{32}

(B) 2^{16}

(C) 2^{8}

(D) 2^{18}

Ans. (B)

#Q. The number of 3-digit numbers which is divisible by 2 and 3 but not divisible by 4 and 9.

36.

6

Total 3-digit nos = 900.

Nos divisible by 6 = $\frac{900}{6}$ = 150.

Nos divisible by 36 = $\frac{900}{36}$ = 25

150 − 25 = 125

999

A

B

C

D

Ans. (125)

↓coefficient

#Q. If the 5th, 6th and 7th term of the binomial expansion of $(1+x^2)^{n+4}$ are in A.P Then the greatest binomial coefficient in the expansion of $(1+x^2)^{n+4}$ is

(A) 10

(B) 35

(C) 25

(D) 14

$^{n+4}C_4,\ ^{n+4}C_5,\ ^{n+4}C_6$

$$2\times\frac{(n+4)!}{5!\,(n-1)!} = \frac{(n+4)!}{4!\,(n)!} + \frac{(n+4)!}{6!\,(n-2)!}$$

$$\frac{2}{5(n-1)} = \frac{1}{n(n-1)} + \frac{1}{6\times 5}$$

$$\frac{2}{5(n-1)} = \frac{30+n^2-n}{30(n)(n-1)}$$

$$12n = 30+n^2-n$$

$n^2 - 13n + 30 = 0$

$(n-3)(n-10) = 0$

$n = 3, 10.$

$n + 4 = 7, 14.$

Greatest B.C. $= {}^7C_3 = 35$

${}^{14}C_7$

Ans. (B)